GDR TRESSES MEETING IN DIJON: ABSTRACTS

1. Mini-courses

Francois Costantino: An introduction to non semi-simple TQFTs

Abstract: After recalling the definition of a TQFT (Topological Quantum Field Theories) and their constructions from manifold invariants, we will detail the differences between the "semi-simple" TQFTs and the "non semi-simple" ones.

We will use as a central example the Hopf algebra $U_q(\mathrm{sl}_2)$ and its different versions.

We will try to explain the topological motivation behind non semi-simple TQFTs by stating a few conjectures on the associated manifold invariants and mapping class group representations.

Julien Marché: Quantum representations of mapping class groups

Abstract: We plan to study those representations through an example that carries a lot of their interesting aspects: the Fibonacci TQFTs (level 5).

We will construct those representations and we will study their arithmetic properties (integrality) as well as geometric properties (action on symmetric spaces).

2. Talks

Léo Bénard: Fried's conjecture for non-unitary representations of unit tangent bundles of 2-dimensional hyperbolic orbifolds

Abstract: Given a compact riemannian manifold M, a vector field with *good* dynamical properties and a unitary representation of the fundamental group of the unit tangent bundle of M, Ruelle defined a zeta function on the set of periodic orbits of this vector field. The statement of Fried's conjecture is that the value of the zeta function of Ruelle at 0 is the modulus of the Reidemeister torsion of the unit tangent bundle

of the manifold in the given representation. This conjecture is a theorem in many cases now, but when the representation is not unitary, still few is currently known.

In the case of the unit tangent bundle of a 2-dimensional hyperbolic orbifolds, we prove that the value of the Ruelle zeta function at 0 is the Reidemeister–Turaev torsion with respect to the Euler structure induced by the geodesic flow.

Joint work with Jan Frahm and Polyxeni Spilioti (Aarhus University).

Fathi Ben Aribi: From paths on Cayley graphs to link invariants

Abstract: Certain operators which act on the completion of a group algebra can be related to random walks on a Cayley graph of the same group. This point of view allows us to compute determinants of such (infinite-dimensional) operators, in the case of free groups, following works of Bartholdi and Dabach-Lalin.

As a consequence, we also establish that a natural function on the braid groups (inspired by the classical Burau formula for the Alexander polynomial) is not invariant under Markov moves, and thus does not yield an invariant of links.

Matthieu Calvez: Strongly contracting elements in Garside groups

Abstract: The additional length graph associated to a Garside group G is a graph whose relation to G is intended to be analogous to the curve graph's relation to the Mapping Class Group. In the case of braid groups, this graph is conjectured to be quasi-isometric to the curve graph of the punctured disk.

We show that the loxodromics are the same for both actions, namely pseudo-Anosov braids. This fact appears as a consequence of the following more general theorem. The axis of every Morse element of a Garside group G (of finite type) is strongly contracting in the Cayley graph of G/ZG w.r.t. the Garside generating set.

This is joint work with Bert Wiest.

Jacques Darné: Nilpotent quandles

Abstract: Quandles are algebraic structures which were introduced independendtly by Joyce and by Matveev in 1982 in order to encode

invariants of knots and links. They showed in particular that the fundamental quandle is a complete invariant of links.

In this talk, we introduce a particular class of quandles, which we call nilpotent. We explore the basic properties of such quandles and of the invariants that they encode. In particular, we show how the language of nilpotent quandles can be used to study invariants of links up to link-homotopy.

Côme Dattin: Wrapped sutured Legendrian homology and the conormal of braids

Abstract: The unit conormal construction takes us from the smooth world to the contact world. In this talk we will show that, if the conormals of two braids (with 2 strands, for a simple case) are Legendrian isotopic, then the braids are equivalent. If time permits we will also sketch some ideas for the general situation.

The main tool will be the wrapped sutured homology, an invariant of Legendrians with boundary, and its associated exact sequence.

Quentin Faes: Triviality of the J_4 equivalence among homology 3-spheres.

Abstract: Considering a homology 3-sphere presented as a gluing of two handlebodies, i.e. by a Heegaard splitting, one can compose the gluing map by any element of the mapping class group $\mathcal{M}$ of the gluing surface. By a Mayer-Vietoris argument, if the gluing map belongs to the Torelli group $\mathcal{I}$, the subgroup acting trivially on the first homology group of the surface, we still get a homology sphere. This so-called Torelli surgeries yield a trivial equivalence relation on homology 3-spheres : any homology 3-sphere can be obtained from S^3 by a sequence of Torelli surgeries.

Somewhat surprisingly, when restricting the maps to be deeper in the Johnson filtration $(J_k)_{k\geq 1}$ of the Torelli group, the equivalence relation seems to stay trivial. It was shown by Morita and Pitsch successively, that homology 3-spheres are always related by J_2 and J_3 surgeries. The latter result was reproved by Massuyeau and Meilhan using finite-type invariants and Habiro's clasper calculus.

In this talk, we use their method to show that J_4 equivalence is also trivial : any homology 3-sphere can be obtained from S^3 by twisting its Heegaard splitting by an element acting trivially on the fundamental group modulo 5-commutators. We will recall and use a formula introduced by Kawazumi and Kuno, as well as the Casson invariant. We

also exhibit some interesting element of the 4-th term of the Johnson filtration.

Vincent Florens: Seifert forms and slice Euler caracteristic of links

Abstract: We define the Witt coindex of a link as a concordance invariant of the Seifert form. We show that it provides an upper bound for the slice Euler caracteristic of the link. This extends in particular the work of Levine on algebraically slice knots.

Joint with S.Orevkov.

Stavroula Makri: Surface braid groups and the Fadell-Neuwirth short exact sequence

Abstract: Surface braid groups were first introduced by Zariski, and generalise the braid groups Bn introduced by Artin in 1925. During the 1960's, Fox introduced an equivalent topological definition for surface braid groups in terms of the fundamental group of configuration spaces.

In this talk we will present the Fadell-Neuwirth short exact sequences of pure braid groups and of mixed braid groups, which is a useful tool in the study of the braid groups and arise from the Fadell-Neuwirth fibration. In particular, we will describe the splitting problem of these sequences, which is closely related to a possible decomposition of these braid group, as the one we have for the pure braid groups of the plane, called the combing operation.

Indeed, such a possible decomposition for the surface braid groups was a central question, during the foundation and the development of the theory of braid groups during the 1960's, studied by Fadell, Fadell-Neuwirth, Fadell-Van Buskirk, Van Buskirk and Birman, among others. For the remaining cases, a complete solution to the splitting problem of the pure surface braid groups was given Gonçalves-Guaschi. Nevertheless, few things are known for the splitting problem of the mixed braid groups and it is still an open problem. In my thesis I have been studying the splitting problem of the mixed braid groups of the projective plane, where I have obtained necessary and in some cases sufficient conditions for which the short exact sequence splits.

Delphine Moussard: A universal finite type invariant for knots in homology 3-spheres

Abstract: In 2004, Garoufalidis and Kricker constructed an invariant of knots in homology 3-spheres, which is a lift of the Kontsevich integral, and they proved that this invariant is a universal finite type invariant for knots whose Alexander polynomial is trivial. I will introduce a refinement of their invariant which is a universal finite type invariant for all knots. Joint work with Benjamin Audoux.

Louis-Hadrien Robert: Categorification of colored Jones polynomial at root of 1

Abstract: Symmetric $\mathfrak{gl}_2$ homology categorifies the colored Jones polynomial. When the construction is done over a field of characteristic p, the homology can be endowed with a pDG structure. This algebraic refinement is an appropriate setting for evaluating q at a pth root of unity on the categorified level. This story will be the topic of the talk. This construction is motivated by the quest of categorification of WRT invariants.

Joint with You Qi, Joshua Sussan and Emmanuel Wagner.

Adrien Rodau: A Slope Invariant and the A-polynomial of knots

Abstract: We present a homological point of view on the logarithmic slope of the A-polynomial of knots related to the $\mathrm{SL}_2(\mathbb{C})$-character variety. This defines a rational function on the $\mathrm{SL}_2(\mathbb{C})$-character variety which unifies various known invariants related to the A-polynomial and the Reidemeister torsion. We also present a method to compute the slope in terms of Alexander matrices and Fox calculus.

Arthur Soulié: On homological representations for surface braid groups and mapping class groups

Abstract: I will describe a general constructions of homological representations for surface braid groups and mapping class groups. This recovers the well-known constructions of Lawrence and Bigelow, and in this sense it unifies these constructions. I will discuss indecomposability and irreducibility of these representations. In particular, any Lawrence-Bigelow representation is indecomposable. I will also present some results on the kernels of the representations associated to the representations of the mapping class groups.

All this represents a joint work with Martin Palmer.

Roland van der Veen: A new construction of quantum character varieties of knots

Abstract: Studying the representations of a knot group into an algebraic group like SL(2) provides interesting algebraic varieties that contain a lot of information about the geometry of knots. When considering the representations up to conjugation these are known as character varieties.

In this talk we will show how Habiro's language of bottom tangles can be used to construct character varieties, improving on our previous work in arxiv1812.09539

As our setting is non-commutative we not only obtain a character variety but also a natural non-commutative deformation. We will give some examples and speculate how our quantization relates to existing ones and quantum knot invariants. This is joint work with Jun Murakami.